Singletons and You

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Preface

Slide available at https://mstksg.github.io/talks/lambdaconf-2017/singletons/singleton-slides.html.

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GHC extensions (potentially) used:

```
{-# LANGUAGE GADTs                 #-}
{-# LANGUAGE KindSignatures        #-}
{-# LANGUAGE LambdaCase            #-}
{-# LANGUAGE RankNTypes            #-}
{-# LANGUAGE RankNTypes            #-}
{-# LANGUAGE ScopedTypeVariables   #-}
{-# LANGUAGE TemplateHaskell       #-}
{-# LANGUAGE TypeFamilies          #-}
{-# LANGUAGE TypeInType            #-}
{-# LANGUAGE TypeOperators         #-}
{-# LANGUAGE UndecidableInstances #-}
```

```
import Data.Kind        -- to get type Type = *
import Data.Singletons
```

Safety with Phantom Types

```haskell
data DoorState = Opened | Closed | Locked
  deriving (Show, Eq)

data Door (s :: DoorState) = UnsafeMkDoor

-- alternatively
data Door :: DoorState -> Type where
    UnsafeMkDoor :: Door s
```

Other similar examples

- State machines (socket connections, file handles, opened/closed)
- Refinement types
- “Tagged” types (santized/unsantized strings)

Phantom types in action

```
closeDoor :: Door 'Opened -> Door 'Closed
closeDoor UnsafeMkDoor = UnsafeMkDoor
```

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closeDoor :: Door 'Opened -> Door 'Closed
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openDoor  :: Door 'Closed -> Door 'Opened
openDoor UnsafeMkDoor = UnsafeMkDoor
```

Phantom types in action

```
doorStatus :: Door s -> DoorState
doorStatus = -- ????
```

We have a problem.

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```haskell
doorStatus :: Door s -> DoorState
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```

More Problems

```haskell
initalizeDoor :: DoorStatus -> Door s
initializeDoor = \case
    Opened -> UnsafeMkDoor
    Closed -> UnsafeMkDoor
    Locked -> UnsafeMkDoor
```

More Problems

```haskell
initalizeDoor :: DoorStatus -> Door s
initializeDoor = \case
    Opened -> UnsafeMkDoor
    Closed -> UnsafeMkDoor
    Locked -> UnsafeMkDoor
```

Neat, but does this work?

More Problems

```
ghci> :t initializeDoor Opened :: Door 'Closed
initializeDoor Opened :: Door 'Closed
```

Oops.

The Fundamental Issue in Haskell

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The Fundamental Issue in Haskell

- In Haskell, types only exist at *compile-time*. They are **erased** at runtime.
- This is a good thing for performance! Types incur no runtime overhead!
- But it makes functions like `doorStatus` fundamentally unwritable without fancy typeclasses.
- . . . or does it?

The Singleton Pattern

```
data SingDS :: DoorStatus -> Type where
    SOpened :: SingDS 'Opened
    SClosed :: SingDS 'Closed
    SLocked :: SingDS 'Locked
```

Creates three constructors:

```
SOpened :: SingDS 'Opened
SClosed :: SingDS 'Closed
SLocked :: SingDS 'Locked
```

The Singleton Pattern

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- There is only one value of type `SingDS 'Opened`, and only one value of type `SingDS 'Closed`.
- The constructor that a `SingDS s` uses reveals to us what `s` is.

The Singleton Pattern

With our new singletons, we can essentially **pattern match** on types:

```
showSingDS :: SingDS s -> String
showSingDS = \case
    SOpened -> "Opened"
    SClosed -> "Closed"
    SLocked -> "Locked"
```

The Singleton Pattern

With our new singletons, we can essentially **pattern match** on types:

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showSingDS = \case
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```

Alone like this, it's a bit boring. We didn't need GADTs for this.

Door Status

```
doorStatus' :: SingDS s -> Door s -> DoorState
doorStatus' = \case
    SOpened -> \_ -> "Door is opened"
    SClosed -> \_ -> "Door is closed"
    SLocked -> \_ -> "Door is locked"
```

- GADT-ness allows us to enforce that the `s` in `SingDS s` is the same as the `s` in our `Door`.

Door Status

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- GADT-ness allows us to enforce that the `s` in `SingDS s` is the same as the `s` in our `Door`.
- Singleton property means that `SingDS s` has a one-to-one correspondence with its constructors.

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- GADT-ness allows us to enforce that the `s` in `SingDS s` is the same as the `s` in our `Door`.
- Singleton property means that `SingDS s` has a one-to-one correspondence with its constructors.
- Pattern matching on that single constructor reveals to us the type of `Door`.

Implicit Passing

```
class SingDSI s where
    singDS :: SingDSI s

instance SingDSI 'Opened where
    singDS = SOpened
instance SingDSI 'Closed where
    singDS = SClosed
instance SingDSI 'Locked where
    singDS = SLocked
```

Implicit Passing

```haskell
class SingDSI s where
    singDS :: SingDSI s

instance SingDSI 'Opened where
    singDS = SOpened
instance SingDSI 'Closed where
    singDS = SClosed
instance SingDSI 'Locked where
    singDS = SLocked

doorStatus :: SingDSI s => Door s -> DoorState
doorStatus = doorStatus' singDS
```

Implicit Passing

```
class SingDSI s where
    singDS :: SingDSI s

instance SingDSI 'Opened where
    singDS = SOpened
instance SingDSI 'Closed where
    singDS = SClosed
instance SingDSI 'Locked where
    singDS = SLocked

doorStatus :: SingDSI s => Door s -> DoorState
doorStatus = doorStatus' singDS

ghci> doorStatus (UnsafeMkDoor :: Door 'Locked)
Door is locked!
```

Initialize Door

```
initializeDoor' :: SingDS s -> Door s
initializeDoor' _ _ = UnsafeMkDoor
```

Initialize Door

```
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initializeDoor' _ _ = UnsafeMkDoor

ghci> :t initializeDoor' SOpened
initializeDoor SOpened :: Door 'Opened
ghci> :t initializeDoor' SClosed
initializeDoor SClosed :: Door 'Closed
```

Initialize Door

Implicit passing style:

```
initializeDoor :: SingDSI s => Door s
initializeDoor = initializeDoor' singDS
```

SingDS vs. SingDSI

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SingDS vs. SingDSI

- Really, `SingDS s ->` is the same as `SingDSI s =>`
- The two are the same way of providing the same information to the compiler, and at runtime.
- We can use the two styles interchangebly.
- One is **explicitly passing the type**, the other is **explicitly passing the type**.

Ditching the phantom

Sometimes we don't care about what the status of our door is, and we want the type system to relax.

Ditching the phantom

Sometimes we don't care about what the status of our door is, and we want the type system to relax.

This is essentially the same as saying that the status of our door is a runtime property that we do not want to (or sometimes can't) check at compile-time.

Ditching the phantom

```
data SomeDoor :: Type where
    MkSomeDoor :: SingDS s => Door s -> SomeDoor
```

Ditching the phantom

```
data SomeDoor :: Type where
    MkSomeDoor :: SingDS s => Door s -> SomeDoor

ghci> let myDoor = MkSomeDoor (initializeDoor SOpened)
ghci> :t myDoor
myDoor :: SomeDoor
ghci> case myDoor of MkSomeDoor d -> doorStatus d
Door is opened.
```

Runtime-deferred types

```
initializeSomeDoor :: DoorStatus -> SomeDoor
initializeSomeDoor = \case
    Opened -> SomeDoor (initialiseDoor' SOpened)
    Closed -> SomeDoor (initialiseDoor' SClosed)
    Locked -> SomeDoor (initialiseDoor' SLocked)
```

Runtime-deferred types

```
initializeSomeDoor :: DoorStatus -> SomeDoor
initializeSomeDoor = \case
    Opened -> SomeDoor (initialiseDoor' SOpened)
    Closed -> SomeDoor (initialiseDoor' SClosed)
    Locked -> SomeDoor (initialiseDoor' SLocked)

ghci> let myDoor = initializeSomeDoor Locked
ghci> :t myDoor
myDoor :: SomeDoor
ghci> case myDoor of MkSomeDoor d -> doorStatus d
Door is locked.
```

The Singletons Library

The singletons library provides a unified framework for creating and working with singletons for different types (not just `DoorStatus`), and also for functions on those types.

http://hackage.haskell.org/package/singletons

The singletons way

```
$(singletons [d|
  data DoorState = Opened | Closed | Locked
    deriving (Show, Eq)
  |])
```

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```
$(singletons [d|
  data DoorState = Opened | Closed | Locked
    deriving (Show, Eq)
  |])
```

This creates three types and three constructors:

```
-- not the actual code, but essentially what happens
data Sing :: DoorState -> Type where
    SOpened :: Sing 'Opened
    SClosed :: Sing 'Closed
    SLocked :: Sing 'Locked
```

Sing is a poly-kinded type constructor (family):

The singletons way

And also

```
instance SingI 'Opened where
    sing = SOpened
instance SingI 'Closed where
    sing = SClosed
instance SingI 'Locked where
    sing = SLocked
```

(`SingI` is a poly-kinded typeclass)

Examples

```
STrue :: Sing 'True
SJust SFalse :: Sing ('Just 'True)
SOpened `SCons` SClosed `SCons` SNil :: Sing '[ 'Opened, '(

ghci> sing :: Sing 'True'
STrue
```

Other stuff created from the library

Some other convenient features:

```
ghci> fromSing SOpened
Opened

ghci> let s = toSing Opened
ghci> :t s
s :: SomeSing DoorStatus
ghci> case s of
        SomeSing SOpened -> "Opened."
        SomeSing SClosed -> "SClosed."
        SomeSing SLocked -> "SLocked."
```

Non-trivial type logic

```
knock :: Door s -> IO ()
knock = -- ??
```

We want to allow the user to knock on a closed or locked door, but not an opened door.

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We can do this simple case using pattern matching, but it's not always feasible or scalable. We want to define a type relationship that can be used by potentially many functions.

Singletons to the Rescue

```
$(singletons [d|
  canKnock :: DoorState -> Bool
  canKnock Opened = False
  canKnock Closed = True
  canKnock Locked = True
  |])
```

Singletons to the Rescue

```
$(singletons [d|
  canKnock :: DoorState -> Bool
  canKnock Opened = False
  canKnock Closed = True
  canKnock Locked = True
  |])

knock :: (CanKnock s ~ True) => Door s -> IO ()
knock _ = putStrLn "knock knock!"
```

Singletons to the Rescue

```
$(singletons [d|
  canKnock :: DoorState -> Bool
  canKnock Opened = False
  canKnock Closed = True
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knock :: (CanKnock s ~ True) => Door s -> IO ()
knock _ = putStrLn "knock knock!"

ghci> knock (initializeDoor SOpened)
Compile Error!!!!
ghci> knock (initializeDoor SClosed)
knock knock!
```

Singletons to the Rescue

```
tryKnock' :: Sing s -> Door s -> IO ()
tryKnock' s = case sCanKnock s of
    STrue  -> knock
    SFalse -> \_ -> putStrLn "Cannot knock door!"

tryKnock :: SingI s => Door s -> IO ()
tryKnock = tryKnock' sing
```

Singletons to the Rescue

```
tryKnock' :: Sing s -> Door s -> IO ()
tryKnock' s = case sCanKnock s of
    STrue  -> knock
    SFalse -> \_ -> putStrLn "Cannot knock door!"

tryKnock :: SingI s => Door s -> IO ()
tryKnock = tryKnock' sing

ghci> tryKnock (initializeDoor SOpened)
Cannot knock door!
ghci> tryKnock (initializeDoor SClosed)
knock knock!
```

Vectors

```haskell
$(singletons [d|
  data N = Z | S N
  |])

data Vec :: N -> Type -> Type where
    VNil :: Vec Z a
    (:*) :: a -> Vec n a -> Vec (S n) a

infixr 5 :*
```

The Types demand it

```
replicateV'
    :: Sing n
    -> a
    -> Vec n a
replicateV' = \case
    SZ   -> \_ -> VNil
    SS n -> \x -> x :* replicateV' n x
```

The Types demand it

```haskell
replicateV'
    :: Sing n
    -> a
    -> Vec n a
replicateV' = \case
    SZ   -> \_ -> VNil
    SS n -> \x -> x :* replicateV' n x

replicateV
    :: SingI n
    => a
    -> Vec n a
replicateV = replicateV' sing
```

Thank you!

- Further confusion: https://blog.jle.im/entry/verified-instances-in-haskell.html